The Measurement Space Solution method

The MSS method makes possible optimal exploitation and representation of the information retrieved from atmospheric soundings, by separating the information coming from the measurements and that obtained from external constraints.

The atmospheric state vector $\mathbf{x}$ is decomposed in its components $\mathbf{x}_a$ and $\mathbf{x}_b$ belonging respectively to the measurement space (the space generated by the rows of the Jacobian matrix of the forward model) and to the null space (the orthogonal complement space):

$$
\mathbf{x} = \mathbf{x}_a + \mathbf{x}_b
$$

where $\mathbf{x}_a$ and $\mathbf{x}_b$ can be determined in terms of the matrices $\mathbf{V}$ and $\mathbf{W}$, whose columns are orthonormal bases of the measurement space and of the null space respectively, and of the projections $\mathbf{a}$ and $\mathbf{b}$ on these orthonormal bases:

$$
\mathbf{x}_a = \mathbf{V}\mathbf{a}, \quad \mathbf{x}_b = \mathbf{W}\mathbf{b}
$$

As detailed in the paper by (Ceccherini et al., 2009) and briefly recalled by the same authors in their contribution to this poster session, the MSS $\mathbf{x}_a$ can be determined using a singular value decomposition, to calculate the $\mathbf{V}$ matrix and to obtain an estimate $\hat{\mathbf{a}}$ of the projection $\mathbf{a}$.

Contributions of the measurement space and null space components of the profile to column

The MSS method can be used to express the contribution to total or partial columns of the components of the profile in the measurement space and in the null space.

The ozone column between altitude levels $z_1$ and $z_2$ can be written in discrete form as the scalar product:

$$
\mathbf{c} = \mathbf{v}^T \mathbf{x}
$$

where we have considered a segmentation of the atmosphere between $z_1$ and $z_2$ in $N$ homogeneous layers of equal thickness $\Delta z$ (with pressure $p_i$, temperature $T_i$ and VMR $x_i$ constant within each layer) and where the components of $\mathbf{v}$ represent the partial columns of air within each layer.

By using the decomposition of $\mathbf{x}$ in its measurement space and null space components, we obtain:

$$
\mathbf{c} = \mathbf{c}_a + \mathbf{c}_b
$$

where $\mathbf{c}_a$ and $\mathbf{c}_b$ are the contributions to the column from the components of the profile in the measurement space and in the null space, respectively:

$$
\mathbf{c}_a = \mathbf{v}^T \mathbf{x}_a = \mathbf{v}^T \mathbf{V}\mathbf{a}, \quad \mathbf{c}_b = \mathbf{v}^T \mathbf{x}_b = \mathbf{v}^T \mathbf{W}\mathbf{b}
$$

An estimate of the components of $\mathbf{c}$ and of the associated contributions to the total error $\epsilon$ is given by:

$$
\hat{\mathbf{c}} = \hat{\mathbf{c}}_a + \hat{\mathbf{c}}_b
$$

Application of the MSS method to data fusion

In the case that two or more instruments sound the same portion of the atmosphere the information contained in the observations of all the instruments can be exploited to obtain a more precise estimation of the ozone column with respect to when a single instrument performs the measurement.

Here we follow the approach to data fusion proposed by (Ceccherini et al., 2009), where starting from the MSIs of the ozone profile of the individual measurements a new MSS is calculated which lies in the union space of the original measurement spaces. Once that this new MSS has been calculated (corresponding to a new matrix $\mathbf{V}$ and a new vector $\mathbf{a}$) the above described procedure can be applied to determine a new estimation of the ozone column that includes the information coming from all the considered measurements.

This approach has been applied to combine the ozone measurements from MIPAS-ENVISAT and IASI-METOP data fusion.

Retrieval of $O_3$ partial and total columns from simulated observations

In Fig. 1, we have reported the percentage difference between the retrieved and the true values of tropospheric, stratospheric and total ozone columns as a function of the number of singular values used for the determination of the measurement space. In the upper panels results of the retrieval from IASI measurements (a), from MIPAS measurements (b) and from IASI/MIPAS data fusion (c) are displayed.

The number of singular values corresponding to the minimum of the total error is highlighted in the upper panels with symbols associated to the specific partial or total columns (triangle for tropospheric, diamond for stratospheric and square for total column).

In Table 1 percentage errors $\epsilon_a$, $\epsilon_b$ and $\epsilon_c$ calculated for the number $p$ of singular values that corresponds to the minimum of the total error.

Sensitivity of the retrieved columns to the true ozone distribution

In order to characterize the sensitivity of the estimated column $\hat{\mathbf{c}}$ to the true ozone distribution and to evaluate its improvement with data fusion, we have considered the averaging kernel (AK) in the partial column space.

This is defined as the vector $\mathbf{A}$, whose $i$th element is the derivative of the estimated column with respect to the partial column $\epsilon_i = \epsilon_i \mathbf{x}$ of the $i$th layer, given by:

$$
A_i = \frac{\partial \hat{c}_i}{\partial \epsilon_i}
$$

The expression above can be derived by writing the derivative of the estimated column with respect to the true ozone VMR profile as:

$$
\frac{\partial \hat{c}}{\partial c} = \mathbf{v}^T \mathbf{A} \mathbf{v}
$$

or, equivalently,

$$
\frac{\partial \hat{c}}{\partial c} = \sum_{i=1}^{N} \Delta z \frac{\partial \epsilon_i}{\partial c_i} A_i v_i
$$

In Fig. 2, we report the averaging kernels $\mathbf{A}$ for the retrieval of tropospheric, stratospheric and total ozone columns, when using:

(a) IASI measurement only,

(b) MIPAS measurement only

(c) IASI and MIPAS data fusion

The AKs of the individual measurements deviate substantially from the ideal behavior (AK equal to 1 within the altitude range of the partial column under consideration and zero outside). The data fusion improves the sensitivity below 6 km w.r.t. MIPAS measurements and above 6 km w.r.t. IASI measurements.

In Fig. 3, we illustrate the dependence of the AKs on the number of considered singular values, showing the AKs of the total column (0-80 km) retrieved for the data fusion of MIPAS and IASI measurements for several values of the number $p$ of considered singular values. For increasing values of $p$ the AK approaches the behavior expected in the ideal case (at expenses of an increase of the total error).

The choice of the optimal $p$ value is driven by the trade-off between the shape of the AK and the

References